

How The Mandelbrot Set is Constructed:

1. One at a time, each point (or computer screen *pixel*) in the complex plane becomes the seed to be iterated through the function:

$$x_{\text{next}} = x^2 + s$$

(where s is the seed)

Example $s = \text{seed} = -1 + .5i$

$$\begin{aligned} x_1 &= (-1 + .5i)^2 + (-1 + .5i) && \text{(constant } s) && = -.25 - .5i \\ x_2 &= (-.25 - .5i)^2 + (-1 + .5i) && && = -1.19 + .75i \\ x_3 &= (-1.19 + .75i)^2 + (-1 + .5i) && && = -1.152 + -1.28i \\ x_4 &= (-1.152 + -1.28i)^2 + (-1 + .5i) && && = -2.62 + .89i \\ & \cdot && && \\ & \cdot && && \\ & \cdot && && \end{aligned}$$

2. For each seed, there are 2 possible **fates** for the **orbit**:
- the orbit will tend to infinity
 - the orbit will not tend to infinity (usually tending toward an attractor)
3. To determine whether an orbit will eventually take off to infinity, you look at the **magnitude** of each iteration in the orbit. ($\|a+bi\|$ = the magnitude of $a+bi$)

For the example above ($s = \text{seed} = -1 + .5i$):

$$\begin{aligned} x_0: & \| -1 + .5i \| && = 1.12 \\ x_1: & \| -.25 - .5i \| && = .56 \\ x_2: & \| -1.19 + .75i \| && = 1.41 \\ x_3: & \| -1.152 - 1.28i \| && = 1.29 \\ x_4: & \| -2.62 + .89i \| && = 2.77 \dots \end{aligned}$$

4. **FACT:** If any iteration of the orbit grows to have a **magnitude greater than 2**, the orbit will approach infinity.
5. How does each point get assigned a **color**?
- If the orbit doesn't tend toward infinity, color the **seed** *black*.
- If the orbit does tend toward infinity, assign the seed a color *based on the number of iterations it took to reach 2*.

For the example above, the orbit took 4 iterations to grow to a magnitude beyond 2. "4 iterations" would have a particular color assigned to it and any other point that took exactly 4 iterations would be assigned the same color.

The **Mandelbrot Set** is the set of black points. The outside of the border of the M-Set is made up of the infinity complex colored regions.